Session 12

Service Systems

Single-Server Queues

Last revised: July 6, 2011

Review of last time: Inventory Modeling

- Inventory modeling is one example of capacity problem
- Inventory is especially important in businesses that deal in materials, and most especially when interest rates are high
- Cost factors associated with inventory include interest expense, ordering cost, space, shrinkage and other holding costs
- When demand is constant, we can define an Economic Order Quantity (EOQ)

Service systems

- Service system: a service facility containing servers, customer entry and exit facilities, and a waiting facility
- Examples
  - Bank tellers/waiting line
  - Airline passenger check-in/waiting line
  - Cafeteria
  - Hospital emergency room
  - Supermarket checkouts
  - Airport runway
  - Highway toll booths
  - Elevators
  - Telephone system (PBX)
  - Gas pumps
  - Tennis courts at health club

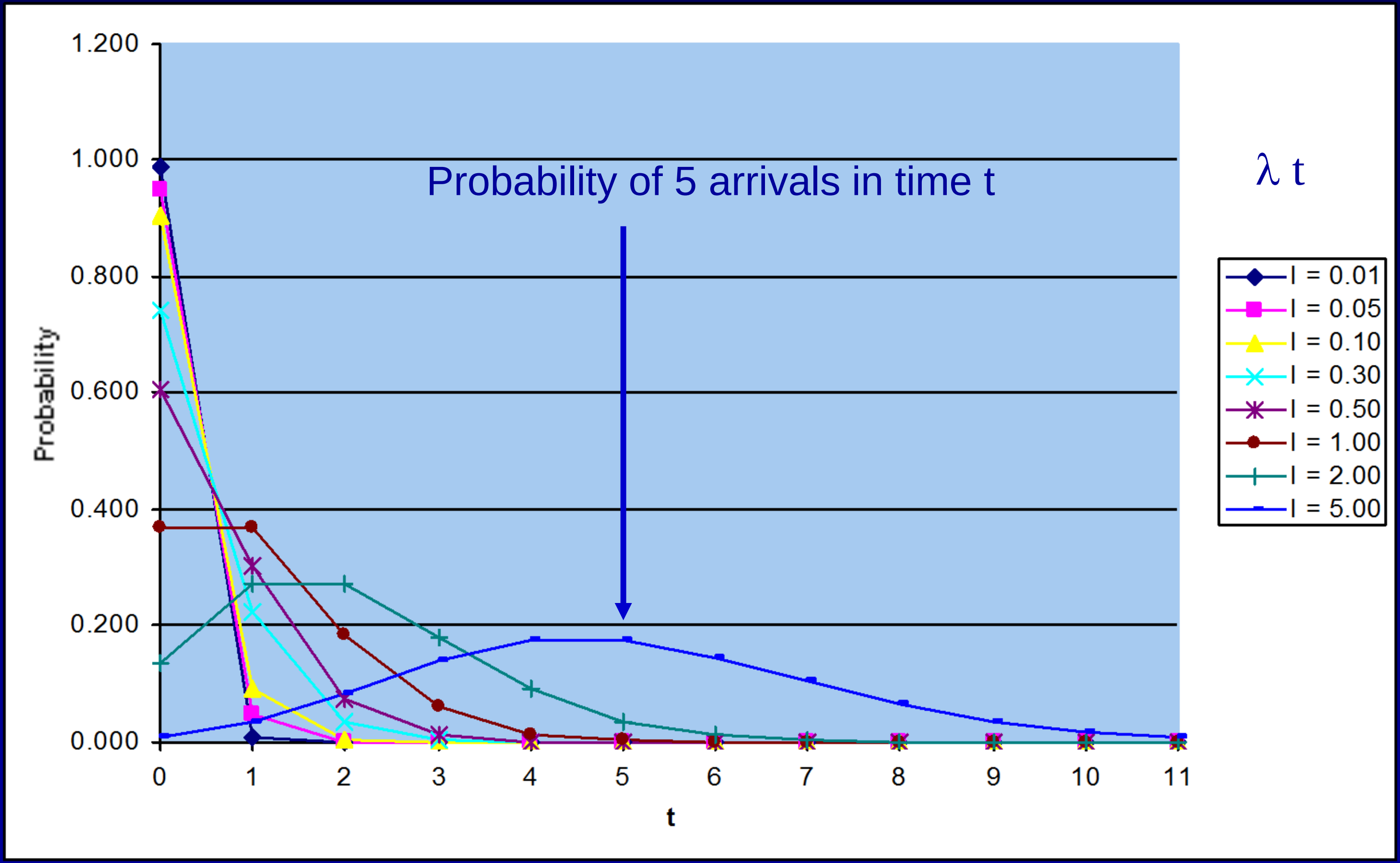
Service system components

- Source population: where customers come from
  - Finite (aircraft to be serviced on a carrier)
  - Infinite, or effectively infinite (Customers for a fast-food outlet)
  - We deal only with infinite populations
- Arrival mechanism: how customers enter the service line
  - Place in line
  - Take a number
- Waiting line or lines: facility for storing customers
- Selection for service: process for selecting next customer
  - FIFO
  - Priority (jump to head of line)
  - Preemptive priority (interrupt customer being served)
- Departure: process for customer exit after service

Arrival characteristics

- Mean arrival rate is constant
- Arrival distribution: statistical distribution of arrival times
  - Batch (e.g., arriving passengers)
  - Random
    - Statistical distribution
    - If arrivals are independent, we model them by Poisson distribution
- Arrival management
  - Often arrivals cannot be controlled
  - Control strategies are used to reduce maximum required server capacity
    - Appointments/scheduling
    - Off-peak pricing discount
    - Peak pricing surcharge

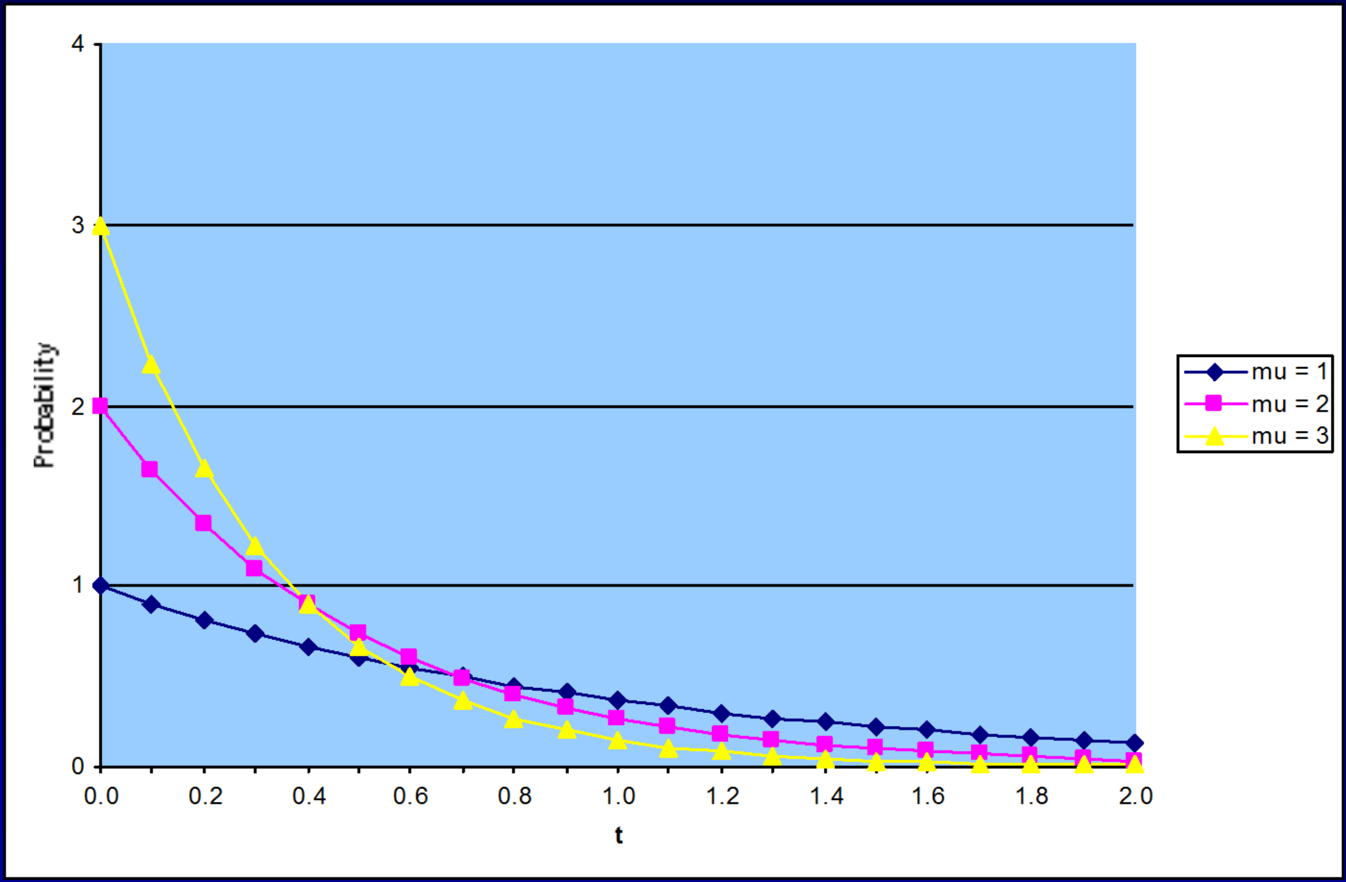
Poisson distribution



Departure characteristics

- Mean service time is constant
- Distribution of service times
  - Random
    - Statistical distribution
    - If service times are independent we model them as the interarrival times of a Poisson Distribution (Exponential Distribution)
- Departure management
  - As soon as a customer is serviced, they instantly depart
  - Inventory security problem
    - Movie theater: separate doors for exit to prohibit multiple viewings
    - Supermarkets
      - Can’t get out without going through a checkout
      - Must leave after checkout
    - Airline check-in: no restrictions on departure after service

Exponential distribution



Example2

Service facility properties

- Service can have stages — single-stage or multistage

  Examples

    - Airport check-in (without kiosk) is single-stage,
      but the passenger terminal is multistage
    - Cafeterias are usually multistage, banks usually single-stage

- Parallelism
  - In servers
  - In waiting area
- Hybrid
  - Tandem toll both is single-stage, single line, multi-server, but the two servers interact

Modeling waiting lines

- Critical assumptions for our model:
  For both arrival and service
  - Rate is constant
    There are two rates — the average arrival rate and the average service rate
  - Arrival events are independent (Poisson distribution)
    The fact that a customer has just arrived doesn't make it any more or less likely that another one will.
    The fact that a customer has just been serviced doesn't change the rate at which the next customer will be serviced.
  - The system is in equilibrium (the doors opened a long time ago)
- Fundamental balance equation
- If P_n = probability that there are *n* customers in the system
  Then

$$\lambda P_{n-1} = \mu P_n$$

Performance measures for single server systems

Probability of *n* customers in the system	$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$	12.1
Utilization of server	$\rho = \frac{\lambda}{\mu}$	12.2
Probability of >*k* customers in the system	$P_{n>k} = \left(\frac{\lambda}{\mu}\right)^{k+1}$	12.3
Average number of customers in the system	$L_s = \frac{\lambda}{\mu - \lambda}$	12.4
Average time in system	$W_s = \frac{1}{\mu - \lambda}$	12.5
Average waiting time	$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$	12.6
Average number waiting	$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$	12.7
Average number waiting when there is a line	$L_a = \frac{\mu}{\mu - \lambda}$	12.8

Example

- Fast food drive-thru
  - Customers arrive at a Poisson rate of 20/hr
  - Service times average 2.6 minutes, exponentially distributed
  - How long is the average waiting line?

◆ Example1

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

  - Excluding empty lines, what is the average line length?

$$L_a = \frac{\mu}{(\mu-\lambda)}$$

Readings

- ➡ Readings: Service Systems

Preview of next time: Using Macros I

- Macros are small programs you write to add capability to Excel
  - Command macros change the environment
  - Function macros compute and return values
- Language: Visual Basic for Applications (VBA)
- VBA Macros reside in modules
- Function macros simplify worksheets (but they require sophistication)