

## Session 3

# Cushioning, Crowding and Quantization

Worksheet Functions  
Circular References  
Implicit Intersection

# Review of last time: Analysis and Synthesis

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- Analysis and Synthesis
  - Decomposing and Recombining
  - Arrays often result — array manipulation required
- Techniques of array manipulation
  - Array arithmetic (+, -, \*, /, ^, ...)
  - Matrix arithmetic (MMULT, TRANSPOSE)
  - Use Matrix arithmetic when you want to perform a synthesis
  - Use Array arithmetic between analysis and synthesis

- Modeling is inexact
  - The validity of a model is based on plausibility, not proof
  - Parameters of a model are either guessed or measured  
In either case, true values are rarely known exactly
  - The system you're modeling is a moving target  
What was once well-understood may evolve out from underneath your model
- Some sources of change (there are many more)
  - Competitive environment
  - Legal environment
  - Technological advances
  - Customer behavior and requirements
  - Your own organization
- For these reasons we often build safety cushions into models

Cushioning is the practice of including safety margins in models to mitigate the effects of approximations and change

- Even when you *can* find an optimum solution:
  - You might still want to a safety margin or cushion
  - You might not know the initial conditions exactly
- Examples
  - Cash reserves greater than the model predicts you need
  - Extra staff, stock or other resources during periods of high volume
  - Leasing more space than needed to allow for faster expansion

Crowding is the opposite of cushioning — you intentionally plan for resources smaller than needed

- Examples
  - Plan for doubling up in offices
  - Planned shortages in computer equipment
  - Planned shortages of staff — planned overtime
  - Planned wages or salaries below prevailing rates — planned turnover
- Usually, this is bad business
- But when you have to do it, it's good to know how

Quantization occurs when resources are available in discrete units, or when attributes change discontinuously

- Sometimes vendors require orders in discrete units
  - No “odd lots”
  - Full trailer load, full roll of carpet, full cases
  - Also for labor: full week, full shift
  - Buildings, vehicles, people are not available in fractional units (except by time-share)
- Minimum quantity requirements
- To get what you need, you must sometimes acquire more (or less) than you need

- The difference between what you actually need and what you plan for is the *planned margin*
- Planned margins
  - Can be additive or multiplicative
  - Can be constant
  - Size can depend on base value, time, or some other quantity
- When the model describes temporal behavior
  - Planned margin can vary with time
  - Planned margin can depend on
    - This period
    - Previous, next, or other period
- When there is dependence on “this period,” watch out for circular reference errors (more on this later)

# Excel functions for cushions, crowding and quantization

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- CEILING (*number, significance*)
  - Returns nearest integer multiple of *significance* on other side of *number* from 0
  - *number* and *significance* must have the same sign
- FLOOR (*number, significance*)
  - Returns nearest multiple of *significance* between 0 and *number*
  - *number* and *significance* must have the same sign
- INT (*number*)
  - Returns most positive integer closest to *number*
- ROUND (*number, numberOfDigits*)
  - Returns multiple of  $10^{-\text{numberOfDigits}}$  that is closest to *number*
- TRUNC (*number, numberOfDigits*)
  - Returns multiple of  $10^{-\text{numberOfDigits}}$  that is closest to *number*, between *number* and 0
- MROUND (*number, base*)
  - Returns *number* rounded to the nearest multiple of *base*
  - *number* and *base* must have the same sign

See On Line Help

◆ ExcelFunctions



# Modeling techniques for cushions

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- You're planning purchase of trucks for a utility company fleet. You want to make sure that you always have enough trucks ready to roll. If the number you need is  $N$ , you can pad  $N$  with:
  - A constant integer  
 $N + \text{Pad}$
  - A computed integer  
 $\text{ROUND}(N * (1 + \text{Pad}), 0)$  gives an integer cushion such that result is a constant percentage greater than the computed value
  - A time-dependent pad  
 $N + \langle \text{reference} \rangle$   
Here we compute the actual value in  $\langle \text{reference} \rangle$

- Spreadsheets calculate the value in a cell by first computing values of cells it depends on
- If this leads back to the original cell, we say there is a “circular reference”
- Most spreadsheets cannot deal with circular references
  - Excel has an iterative recalculation mode
  - Generally, performance is so poor that you should avoid using it
- Circular reference errors are more frequent when the cushioning or crowding factor depend on the previous or next period

- Simple case:
  - $Y = 3 - X$
  - $X = 1 + Y$
- These are simple, linear simultaneous equations
- The solution is  $X = 2, Y = 1$
- If you implement  $X$  and  $Y$  as cells, Excel cannot resolve the solution because you get a circular reference error
- Best approach: solve the equations, then implement that solution

# Unwinding circular references, continued

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- In most common cases, dependencies are linear:

$C_{OH}$  Cash on hand

$C_o$  Cash before interest expense

$D_N$  New debt

$D_{CP}$  Cumulative prior debt

$C_{MIN}$  Cash minimum

$i$  Interest rate per period

If new debt is required: 
$$C_{OH} = C_o - i(D_N + D_{CP}) + D_N = C_{MIN}$$

$$D_N = C_{MIN} - (C_o - i(D_N + D_{CP}))$$

If new debt is not required:

$$D_N = 0$$

$$C_{OH} = C_o - i(D_N + D_{CP})$$

To eliminate circularity, solve equations symbolically

- Ordering fleet vehicles with various planned margin strategies
  - ◆ Example1
- Simple circularity
  - ◆ Example2
- Diagram of a more complex circularity
  - ◆ Example3
- Minimum cash on hand with circularity
  - ◆ Example4

- Excel interprets non-array formulas that refer to ranges using a technique called “implicit intersection”
- If the formula is an array formula, the interpretation proceeds normally.
- Otherwise Excel retrieves the value of the leftmost (topmost) cell in the independent range that is in the same row (column) as the dependent cell
- If there is no such cell, a #VALUE error occurs



ImplicitIntersection

# Preview of next time: Temporal Response

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- Excel functions for accessing array elements:  
`INDEX`, `OFFSET`
- References can be combined to produce new references using reference operators: Colon, Comma and Space
- Time evolution and temporal response model business processes
  - Convolution applies when system behavior is time-independent and additive
  - The Convolve macro implements convolution
  - Also implement convolution with tables

Special Note: Read the session notes for Session 4 to prepare