Session 9

Capital Leases I: Present and Future Value

Worksheet Functions
Non-Uniform Payments

Last revised: July 6, 2011

Review of last time: Financial Models

- Three external financial statements
  - Income Statement (Revenue and Expenses for a given period)
  - Balance Sheet (Financial Position at the end of the period)
  - Cash Flow (Statement of Changes in Financial Position during the period)
- What are capital transactions?
  - Either purchases or sales
  - They have effects beyond the current reporting period
- Capital transactions affect all three financial statements
  - Income Statement
    Depreciation, Maintenance
  - Balance sheet
    Value of the equipment is added to assets
  - Cash Flow
    If purchased for cash, the purchase affects cash on hand

Capital leases

- A Capital Lease:
  - Long-term, non-cancelable contract
  - Transfers most risks and rewards of ownership from lessor to lessee
  - Lessee assumes most costs of ownership
    - Taxes
    - Insurance
    - Maintenance
  - Lessee usually pays a periodic, uniform payment
- To understand the impact of a capital lease, we need to explore two concepts:
  - Present Value
  - Future Value
- We'll discuss these two items now, then return to Capital Leases in our next session

Present and future value

- Both concepts are intimately related to the time value of money
  - A sum of money that you receive a year from now is worth less to you now than an equal sum that you receive immediately
  - The difference in value of these two sums is due to the difference in times when you receive them
  - The amount of the difference is equal to the amount that can be earned from investing the sum you receive now
- The "future value" of an asset A at a future time T is equal to A plus what it can earn at interest by time T
- The "present value" of an asset A to be received at time T is the value now of an asset whose future value at time T is A

Future value

- You receive $100. Prevailing interest rate is 4%, simple interest. What is the future value of the asset in 1 year?

  Interest = Principal * Rate * Time

  Principal + Interest = Principal + Principal * Rate * Time
  = Principal * (1 + Rate * Time)
  = 100 * (1 + 4% * 1)
  = 104

- You receive $100 per month for one year. The interest rate is 4% annually, compounded monthly. What is the future value of the asset in one year?

  $100 * (1 + 4\%/12)^{12}$
  $+ 100 * (1 + 4\%/12)^{11}$
  $+ 100 * (1 + 4\%/12)^{10} + \ldots$

Future value in Excel

- Excel has a future value function (a worksheet function) that solves this problem easily: `FV`
- Use the `FV` function for computing future value at compound interest for:
  - A lump sum at T=0
  - A stream of constant payments (annuity or installment)
  - Both
- Interest is compounded per period
- `FV`(*rate, nper, pmt, pv, type*)
  - rate: Interest rate per period
  - nper: Number of periods
  - pmt: periodic payment
  - pv: present value or starting amount
  - type: If 0: payments are made at the end of a period (default)
If 1: payments are made at the beginning of a period

Future value in Excel (continued)

- Why the minus sign?
  - In FV(*rate,nper,pmt*) the result has a sign opposite to the sign of *pmt* Why?
  - In FV(*rate,nper, ,pv*) the result has a sign opposite to the sign of *pv* Why?
- FV assumes that when you make a series of payments, you're paying out a stream in order to receive a value in return at a future date

  - You save $2,000/year for 35 years at 4% per year compounded annually. You make payments at year-end. How much will you have at the end of 35 years?
- FV assumes that when you pay a lump sum now, you're paying it in order to receive a value in return at a future date.

  - You put aside $10,000 now at 4% per year compounded annually. How much will you have in 35 years?

Future value examples

- You're saving for retirement. You've found an investment vehicle that pays 4% compounded annually. You now have $10,000. If you add $2,000 at the beginning of every year, what will be the value of your savings in 35 years?

  C `=FV(4%,35,-2000,-10000,1)`

- How much will you have if, in years 21 through 35, the interest rate is 6%?

  D `=FV(6%,15,-2000,-FV(4%,20,-2000,-10000,1),1)`

- How much will you have if, in years 21 through 35, you add $2,500 per year but the interest rate remains at 4%?

  E `=FV(4%,15,-2500,-FV(4%,20,-2000,-10000,1),1)`

Non-uniform interest or payments require careful treatment

Non-uniform future value

- Future value with non-uniform payments can be done period-by-period. Technique relies on ***linearity***:

  The future value of a sum is the sum of the future values

- Example:
  - Interest over a 4-year period is 4% (compounded annually)
  - Payments are 2,500; 2,500; 3,200; 4,200, made at the beginning of each period.
  - How much is available at the end of year 4?

F

- When payments are made at the beginning of the period, `FV` is equivalent to `CONVOLVE`
  - `CONVOLVE` may be more convenient for non-uniform payments
  - Interest rate must be constant

G

Present value

- Prevailing interest rate is 4%, simple interest. How much do you have to have invested to achieve a total value of $100 in 1 year?

  Principal + Interest = Principal + Principal * Rate * Time
  = Principal * (1 + Rate * Time)
  = Principal * (1 + 4% * 1)
  = 100

  Principal = 100/1.04

- The interest rate is 4%, compounded monthly. To reach a total future value of $100 in one year, how much must you save every month?

  $X * (1 + 4\%/12)^{12}$
  $+ X * (1 + 4\%/12)^{11}$
  $+ X * (1 + 4\%/12)^{10} + \ldots = 100$

Present value in Excel

Excel has a present value function (a worksheet function): PV

- Use the `PV` function for computing present value at compound interest of:
  - A lump sum at some future time
  - A stream of constant payments (annuity or installment)
  - Both
- Interest is compounded per period
- `PV`(*rate, nper, pmt, fv, type*)
  - rate: Interest rate per period
  - nper: Number of periods
  - pmt: periodic payment
  - fv: future value or goal
  - type: If 0: payments are made at the end of a period (default)
If 1: payments are made at the beginning of a period

Present value examples

- You're saving for retirement. You've found an investment vehicle that pays 4% compounded annually. Your goal is $500,000. How much must you invest now to reach your goal in 35 years?

H

```
=PV(4%,35,0,-500000,0)
```

- How much do you need if, in years 21 through 35, the interest rate is 6%?

I

```
=PV(4%,20,0,-PV(6%,15,0,-500000,0),0)
```

- You win $1M in the lottery. Assuming that the interest rate is 4%, would you rather have $50K each year for 20 years or $750K in a lump sum right now?

J

```
=PV(4%,20,-50000,0)
```

Non-uniform present value

- Present value of a non-uniform payment stream can be done period-by-period.
- Example:
  - Interest over a 4 year period is 4% (compounded annually)
  - Payments are 2,500; 2,500; 3,200; 4,200, at the beginning of each period
  - What is the present value of this income stream?

Calculating payments

- Excel has a worksheet function for computing the size of payments required to reach a specified future value: PMT
- PMT(*rate*, *nper*, *pv*, *fv*, *type*)
  - rate: Interest rate per period
  - nper: Number of periods
  - pv: total amount that the payments are worth now
  - fv: future value or goal
  - type: If 0: payments are made at the end of a period
If 1: payments are made at the beginning of a period
- PMT is related to FV and PV, for a given rate and nper
- Two other worksheet functions:
  - IPMT: the interest portion of the payment
  - PPMT: the principal portion of the payment

Payment calculation examples

- How much do you have to save each month in a savings account that pays 4% per year compounded monthly to reach a goal of $100 in 10 months?
- What is the future value of that payment stream?
- What's the present value of that future value?
- How much would you have to pay each month to repay a loan of that amount if the interest rate is 4% per year, compounded monthly?

Reference readings

- On line help topic:
  Index: "worksheet functions, by category";
  see category "Financial"

  Find entries for FV, IPMT, PMT, PPMT, PV
- Walkenbach, John. *Excel 2003 Bible*. John Wiley & Sons, 2003.
- Walkenbach, John. *Excel 2007 Bible*. John Wiley & Sons, 2007.
- Walkenbach, John. *Excel 2010 Bible*. John Wiley & Sons, 2010.
- Welch, Anthony, and Short. *Fundamentals of Financial Accounting*. Homewood, IL: Irwin.
- Robert Libby, Patricia Libby and Daniel G. Short. *Financial Accounting (Fourth Edition)*. McGraw-Hill/Irwin, 2003.

Preview of next time: Capital Leases II: Modeling

- Capital leases affect all three financial statements
  - Cash outlays
  - Assets
  - Liabilities
  - Depreciation
  - Interest expense
- For streams of lease events under identical leases
  - Create a Lease Characteristic Array (LCA)
  - Convolve LCA with lease event stream
- In models with multiple identical leases, combine lease streams, then compute effects on financial statements rather than vice versa