Session 11

Inventory Modeling

Economic Order Quantity
Quantity Discounts

Last revised: July 6, 2011

Review of last time: Capital Leases II: Modeling

- Capital leases affect all three financial statements
  - Cash outlays
  - Assets
  - Liabilities
  - Depreciation
  - Interest expense
- For streams of lease events under identical leases
  - Create a Lease Characteristic Array
  - Convolve with lease event stream
- In models with multiple identical leases, combine lease streams, then compute effects on financial statements rather than vice versa

Capacity models

- Categories of capacity questions
  - Inventory
    - "Do we have enough"
    - "What's the restock point?"
    - "What is the best choice of restock quantity?"
  - Servers
    - "How many checkouts should a supermarket have open?"
    - "How many phone lines do we need to support our business?"
    - "How many customer service reps are enough?"
    - "How many file servers are enough?"
  - Space
    - "When should we lease more?"
    - "How much parking do we really need?"
- Each category of capacity problem requires a unique approach.

Importance of inventory

- Inventory management is most important in goods-oriented operations
  - Manufacturing
  - Mining
  - Retail
  - Real estate development/rental
  - Wholesale distribution/logistics
  - Fulfillment
- Inventory is less important when
  - Added value is mostly not related to goods
    - Professional services operations
    - Software or other knowledge-driven companies
  - Finished Goods (and their components) cannot be stocked
    - Electric power
    - Broadcasting
    - Telecommunications
    - Common carrier transport
    - Education
    - Entertainment
  - But even these organizations must inventory some items
- Keep inventory low when holding costs are high

Kinds of inventory

- Raw materials and purchased parts for use in finished goods
- Work in process (partial assemblies)
- Finished goods
- Maintenance inventory
- Internally generated vs. externally acquired inventory

Inventory cost factors

- Costs related to the number of orders
  - Decision to order
  - Telephone, postage, other communications
  - Labor
  - Computation and record-keeping (accounting, tracking, control)
  - Price verification (external only)
  - Receiving and handling
  - Freight
  - Setup and stowage
- Costs related to the total value and bulk of inventory
  - Handling and storage
  - Interest costs
  - Opportunity costs relative to alternative inventoried components
  - Insurance, taxes, space, equipment
  - Heating, cooling, humidity control
  - Shelf life factors: spoilage, theft, obsolescence

Inventory cost factors (2)

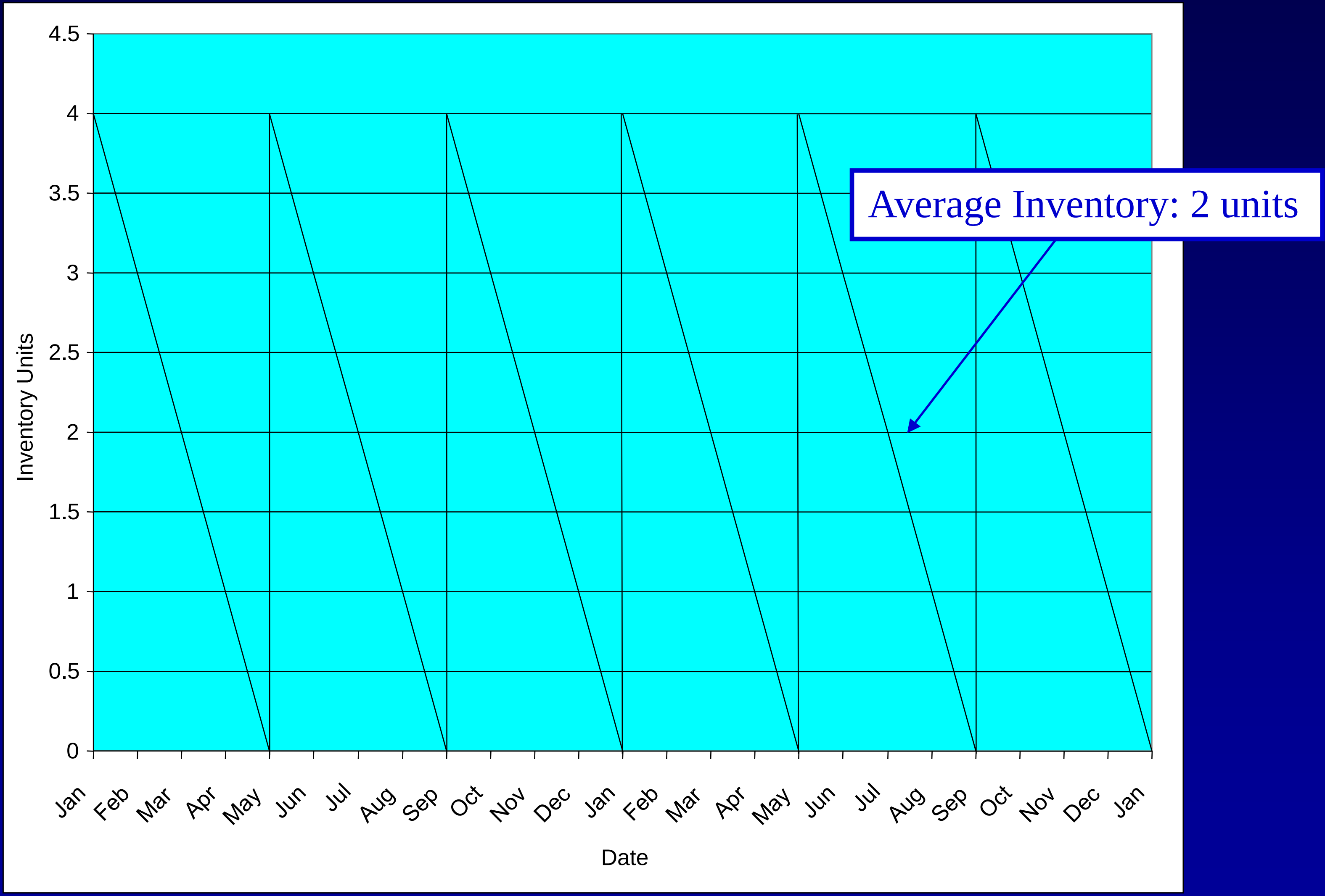
- Costs of stockouts
  - Lost sales
  - Lost customers
  - Idle time (no parts -> idle assembly lines)
  - Idle workpieces in process
  - back-order/expedite cycle
- Size of order
  - quantity discounts
  - freight
  - insurance
  - security

Knowing the true cost of inventory is important even when you outsource it.

Effects of demand On inventory strategy

- Constant demand vs. variable demand
  - Constant demand is easier to manage
  - Constant demand is easier to model
  - Constant demand is more and more rare because:
    - The world is changing rapidly
    - Repeated orders of identical products are less and less common
- Variable demand is best modeled statistically
- First understand the constant demand problem

Representing constant demand



Constant demand data

- This is the data for the graphical example on the previous slide
- Inventory is replenished every fourth month
- The data show inventory at the start of each month
- Notice: no zeros in the table, but the restock point *is* zero.
- Average inventory for constant demand:

$$I_{average} = \frac{I_{reorder} + I_{max}}{2}$$

- If $I_{reorder} = 0$:

$$I_{average} = \frac{I_{max}}{2}$$

Jan	4
Feb	3
Mar	2
Apr	1
May	4
Jun	3
Jul	2
Aug	1
Sep	4
Oct	3
Nov	2
Dec	1
Jan	4
Feb	3
Mar	2
Apr	1
May	4
Jun	3
Jul	2
Aug	1
Sep	4
Oct	3
Nov	2
Dec	1
Jan	0

Constant demand

For constant demand, use evenly spaced, equal sized orders

On an annual basis:

$$\begin{pmatrix}\text{Total}\\ \text{Cost}\end{pmatrix} = \begin{pmatrix}\text{Cost Related}\\ \text{to No. Orders}\end{pmatrix} + \begin{pmatrix}\text{Cost Related to}\\ \text{Inventory Size}\end{pmatrix}$$

$$= \frac{D}{Q} C_0 + \frac{Q}{2} C_c$$

$$= \frac{D}{Q} C_0 + \frac{I_{max}}{2} C_c \qquad \text{(if restock point is 0)}$$

where

D = Annual demand (units per year)

Q = Quantity in one order (units)

C_0 = Cost to place one order (dollars)

C_c = Carrying cost of inventory (dollars per unit per year)

Economic order quantity

- EOQ: Lowest-cost order quantity for constant demand

$$\sqrt{\frac{2DC_0}{C_c}}$$

It can be shown mathematically that EOQ is the value of Q for which Ordering Costs = Carrying Costs.

➡ Readings: Economic Order Quantity

Quantity discounts

When there is a quantity discount, there is an additional term in the cost equation:

$$\text{Total Cost} = \frac{D}{Q} C_0 + \frac{Q}{2} C_c + D\, u(Q)$$

u(Q) = unit price as a function of order size

Here we are computing not only the inventory costs, but the buying costs as well.

To minimize total costs, you have to take an experimental approach: compute costs for each break in the quantity schedule.

Examples

- You are a buyer of dyes for an apparel manufacturer. The annual demand for dyes of all colors is 64,000 liters. Price is $10 per liter. The cost to hold an item in inventory is 20% of the value of the item per year. It costs $10 to place an order (you use email). What is the EOQ? Make plots showing the two components of total cost as functions of order quantity.
- You expect volume to pick up next year, so you negotiated a quantity discount. If you buy more than 1000 liters at a time, you can get a price of $8 per liter. If you buy more than 3000 liters at a time, the price is only $7 per liter. Now what is the optimum order quantity?

◆ Example

Readings

- ➡ Readings: Economic Order Quantity

Preview of next time: Service Systems

- A Service System is a facility containing servers, customer entry and exit facilities, and a waiting facility
- The simplest system is single-stage, single-server
- When arrivals are Poisson distributed, and service completion is exponentially distributed, we can predict system performance